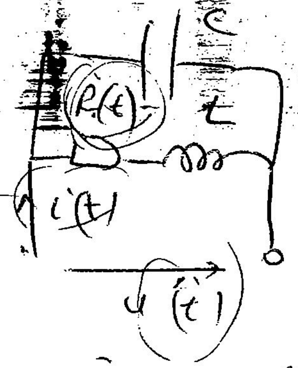
1) Se consideră circuitul cu parametri variabili în timp



Să se determine condiția ca $i(t)$ să fie un semnal modulat în fază pentru $u(t)$ cu variație armonică

$\frac{1}{\cancel{s}}$ sC

2) Se dă funcția de transfer

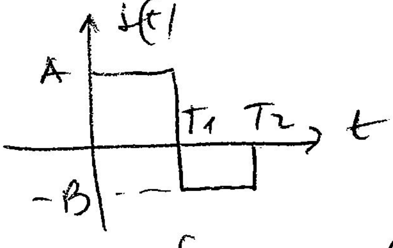
$$A(z) = \frac{1-d_1^* z}{z-d_1}$$

a) Să se determine proprietățile răspunsului în frecvență ale funcției

b) să se determine proprietățile funcției

$$\left(1-|A(z)|^2\right) \text{ pentru } |z|^2 \gtreqless 1$$

3) Să se determine condiția ca răspunsul sistemului cu $H(s)=\frac{\alpha}{s+\alpha}$ la semnalul

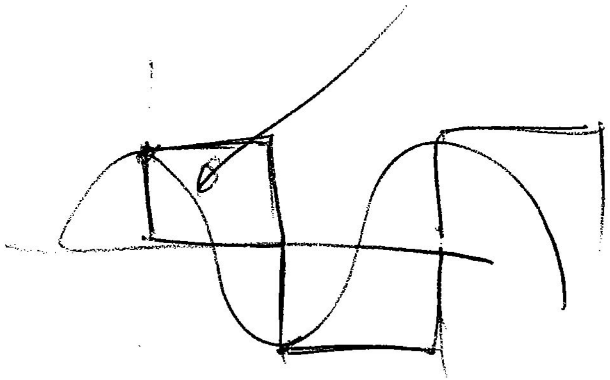


să se anuleze pentru $t>T_2$

4) Să se calculeze

$$\int_{-\infty}^{\infty} \frac{\sin\omega\tau}{\omega\tau}\sin\omega\frac{\tau}{2}\left(e^{j\omega\frac{\tau}{2}}-e^{-j\omega\frac{\tau}{2}}\right)d\omega$$

5) Un semnal MIA$_r$ este obținut prin eșantionarea unui semnal armonic. Determinați condiția ca armonicele a II-a a semnalului periodic obținut să fie nule iar raportul dintre armonica I și III să fie 1/3.



Concurs Tănăsescu
Faza Locală 2006

2006

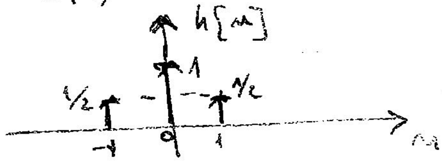
Cleju Nicolae

(1). a). $y[n] = \frac{1}{2} e[n+1] + e[n] + \frac{1}{2} e[n-1]$

$$Y(z) = \frac{1}{2} z \cdot E(z) + E(z) + \frac{1}{2} E(z) z^{-1}$$ 10

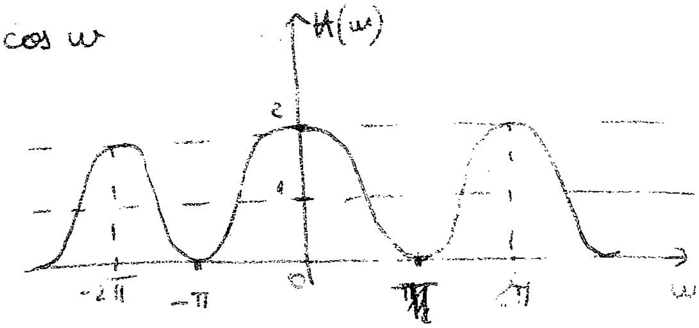
$$Y(z) = E(z)\left(\frac{1}{2}z + 1 + \frac{1}{2}z^{-1}\right) \Rightarrow H(z) = \frac{Y(z)}{E(z)} = \frac{1}{2}z + 1 + \frac{1}{2}z^{-1}$$

$$\Rightarrow h[n] = \frac{1}{2}\delta[n+1] + \delta[n] + \frac{1}{2}\delta[n-1]$$



$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{1}{2}e^{j\omega} + 1 + \frac{1}{2}e^{-j\omega} = \frac{1}{2}\left(e^{j\omega} + e^{-j\omega}\right) + 1 =$$

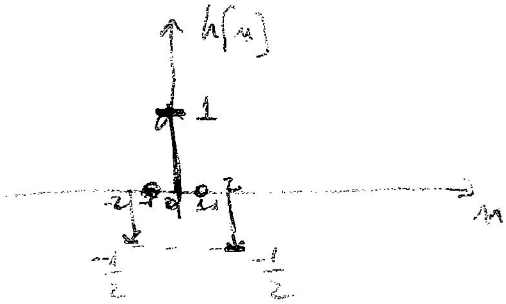
$$= \frac{1}{2} \cdot 2\cos\omega + 1 = 1 + \cos\omega$$



b). $y[n] = -\frac{1}{2} e[n+2] + e[n] - \frac{1}{2} e[n-2]$

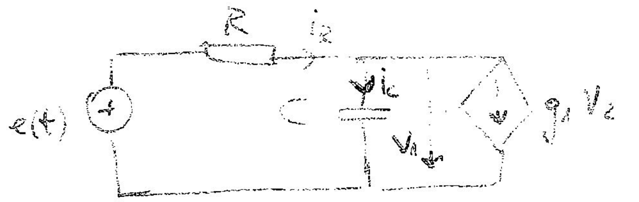
$$Y(z) = -\frac{1}{2} z^2 E(z) + E(z) - \frac{1}{2} z^{-2} E(z) \Rightarrow H(z) = -\frac{1}{2}z^2 + 1 - \frac{1}{2}z^{-2}$$

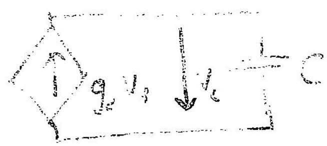
$$\Rightarrow h[n] = -\frac{1}{2}\delta[n+2] + \delta[n] - \frac{1}{2}\delta[n-2]$$



$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} =$$

(2)





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$$e(t) = E_{max} \cdot \cos w_0 t$$

$$V_2 = \frac{1}{C} \cdot \int_0^t g_2 V_1(t)\, dt = \frac{g_2}{C} \cdot \int_0^t V_1(t)\, dt$$

$$i_C = C \cdot \frac{d V_1(t)}{dt} \Rightarrow i_R = i_C + g_1 V_2 = C \cdot \frac{d V_1(t)}{dt} + \frac{g_1 \cdot g_2}{C} \int_0^t V_1(t)\, dt$$

<u>In Laplace</u>:

$$\left.\begin{aligned} I_R(s) &= C \cdot s \cdot V_1(s) + \frac{g_1 \cdot g_2}{C} \frac{1}{s} V_1(s) \\ V_1(s) &= E(s) - R \cdot I_R(s) \end{aligned}\right\} \Rightarrow$$

$$V_1(s) = E(s) - RC\, s V_1(s) - \frac{R g_1 g_2}{C} \cdot \frac{1}{s} V_1(s) \Rightarrow$$

$$\Rightarrow V_1(s) \left(1 + RCs + \frac{R g_1 g_2}{C} \cdot \frac{1}{s} \right) = E(s) \Leftarrow$$

$$V_1(s) \left(RC s^2 + s + \frac{R g_1 \cdot g_2}{C} \right) = s \cdot E(s)$$

$$V_1(s) = \frac{s}{RCs^2 + s + \frac{R g_1 g_2}{C}} \cdot E(s) = \frac{1}{RC} \cdot \frac{s}{s^2 + \frac{1}{RC} s + \frac{g_1 g_2}{C^2}} E(s)$$

$$\frac{V_1(s)}{E(s)} = H(s) = \frac{1}{RC} \cdot \frac{s}{s^2 + \frac{1}{RC} s + \frac{g_1 g_2}{C^2}} \Rightarrow H(\omega) = \frac{1}{RC} \cdot \frac{j\omega}{-\omega^2 + \frac{1}{RC} j\omega + \frac{g_1 g_2}{C^2}} \Leftarrow$$

$$V_1(t) = E_{max} \cdot |H(\omega_0)| \cdot \cos\left(\omega_0 t + \angle H(\omega_0)\right)$$

$$\Rightarrow V_{1max} = E_{max} \cdot |H(\omega_0)| = E_{max} \cdot \frac{1}{RC} \cdot \frac{\omega_0}{\sqrt{\left(\omega_0^2 - \frac{g_1 g_2}{c^2}\right)^2 + \frac{1}{R^2c^2}\omega_0^2}} \Rightarrow$$

$$\left([g] = \frac{A}{V} \Rightarrow \frac{A^2}{\cancel{V^2} \cdot \frac{C^2}{\cancel{V^2}}} = \frac{A^2}{C^2} = \frac{C^2/\cancel{s^2}}{C^2} = \frac{1}{s^2} \; A \qquad [C] = \frac{C}{V} \qquad [\omega_0]^2 = \frac{1}{s^2} \right)$$

• ~~V_{1max} e maxim când~~ $\left\{ \begin{array}{l} \frac{g_1 g_2}{c^2} = \omega_0^2 \\ \cancel{R^2C^2 = \text{maxim}} \end{array} \right.$ ~~$\Rightarrow \frac{g_1 \cdot g_2}{c^2} = \omega_0^2$, R = cât mai mare~~

$$V_{1max} = E_{max} \cdot \frac{\omega_0}{\sqrt{R^2C^2\left(\omega_0^2 - \frac{g_1 g_2}{c^2}\right)^2 + \omega_0^2}} = \frac{\omega_0}{\sqrt{\left(RC\omega_0^2 - \frac{R g_1 g_2}{c}\right)^2 + \omega_0^2}} \cdot E_{max}$$

V_{1max} e maxim când $\cancel{R}C\omega_0^2 = \frac{\cancel{R}g_1 g_2}{c} \Rightarrow \omega_0^2 = \frac{g_1 g_2}{c^2}$

$$\Rightarrow (V_{1max})_{max} = \frac{\frac{\sqrt{g_1 g_2}}{c}}{\sqrt{\frac{g_1 g_2}{c^2}}} \cdot E_{max} \cdot \cancel{1} = E_{max}$$

(3) $f(n) = a^{|n|}, \quad |a| < 1$

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$$F(z) = \sum_{n=-\infty}^{\infty} a^{|n|} \cdot z^{-n} = \sum_{n=0}^{\infty} a^{n} \cdot z^{-n} + \sum_{n=1}^{\infty} a^{n} z^{n} =$$

$$= \sum_{n=0}^{\infty} (a \cdot z^{-1})^{n} + \sum_{n=0}^{\infty} (az)^{n} - 1 = \frac{1}{1-az^{-1}} + \frac{1}{1-az} - 1$$

pentru $|az^{-1}| < 1 \Leftrightarrow |a| \cdot |z|^{-1} < 1$

și pentru $|az| < 1 \Leftrightarrow |a| \cdot |z| < 1$

$$\sum_{n=0}^{\infty} (az^{-1})^{n} = \frac{1}{1-az^{-1}} \quad \text{pt. } |az^{-1}| < 1 \Leftrightarrow |a| \cdot |z|^{-1} < 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{|z|} < \frac{1}{|a|} \Rightarrow |z| > |a|$$

și neconverg. în rest

$$\sum_{n=0}^{\infty} (az)^{n} = \frac{1}{1-az} \quad \text{pt } |az| < 1 \Leftrightarrow |a| \cdot |z| < 1 \Leftrightarrow |z| < \frac{1}{|a|}$$

și neconverg. în rest

$$\Rightarrow F(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-az} - 1, \quad RC: |a| < |z| < \frac{1}{|a|}$$

· $f_1(t) = e^{-\alpha|t|}$ $\quad f_1[n] = T \cdot f_1(nT) = T \cdot e^{-\alpha|nT|} = T \cdot e^{-\alpha|n| \cdot T}$

$$F_1(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} f_1[n] \cdot e^{-j\omega nT} = \sum_{n=-\infty}^{\infty} T \cdot e^{-\alpha|n| \cdot T} \cdot e^{-j\omega nT} =$$

$= \sum T \cdot e^{-\alpha nT} \cdot e^{-j\omega nT} + \sum T \cdot e^{\alpha nT} \cdot e^{j\omega nT}$ [illegible]

$= T \cdot \sum$ [illegible]

$$= T\cdot\frac{1}{1-e^{-\alpha T-j\omega T}} + T\cdot\frac{1}{1-e^{-\alpha T+j\omega T}} - T \quad , \quad pt.\ \alpha > 0$$

SAU :

$$f_1[n] = T\cdot e^{-\alpha|n|\cdot T} = T\cdot\left(e^{-\alpha T}\right)^{|n|}$$

$$F_1(z) = T\cdot\frac{1}{1-e^{-\alpha T}z^{-1}} + T\cdot\frac{1}{1-e^{-\alpha T}z} - T \Rightarrow \cancel{F(e^{j\omega T}) = T} \quad , \quad R.C.:\ |a| < |z| < \frac{1}{|a|}$$

$$a = e^{-\alpha T}$$

$$\Rightarrow F_1(e^{j\omega T}) = T\cdot\frac{1}{1-e^{-\alpha T-j\omega T}} + T\cdot\frac{1}{1-e^{-\alpha T+j\omega T}} - T = \text{identic ca mai sus}$$

$$\text{pentru } \left|e^{-\alpha T}\right| < 1 < \frac{1}{\left|e^{-\alpha T}\right|} \Leftrightarrow$$

$$\Leftrightarrow \left|e^{-\alpha T}\right| < 1 \Leftrightarrow e^{-\alpha T} < 1 \Leftrightarrow e^{\alpha T} > 1$$

$$\Leftrightarrow \alpha T > 0 \Leftrightarrow \underline{\alpha > 0}$$

Pt. $\alpha < 0$, $f_1(t)$ și $f_1[n]$ sunt nemărginite, deci nu au transformată Fourier

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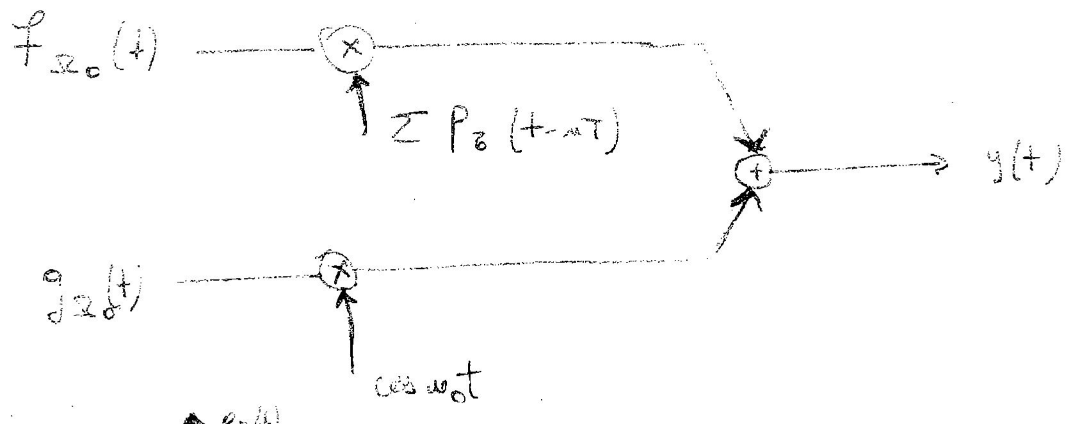

$p_\tau(t) =$

$y(t) = f_{\Omega_0}(t)\cdot \sum p_\tau(t-nT) + g_{\Omega_0}(t)\cdot \cos\omega_0 t$

$Y(\omega) = \frac{1}{2\pi} F(\omega) * \mathcal{F}\{\Sigma\, p_\tau(t-nT)\}$ [illegible]

$$\mathcal{F}\left\{ \sum p_\tau(t-nT) \right\} = \mathcal{F}\left\{ p_\tau(t) * \delta_T(t) \right\} = P_\tau(\omega) \cdot \Omega\, \delta_\Omega(\omega), \qquad \Omega = \frac{2\pi}{T}$$

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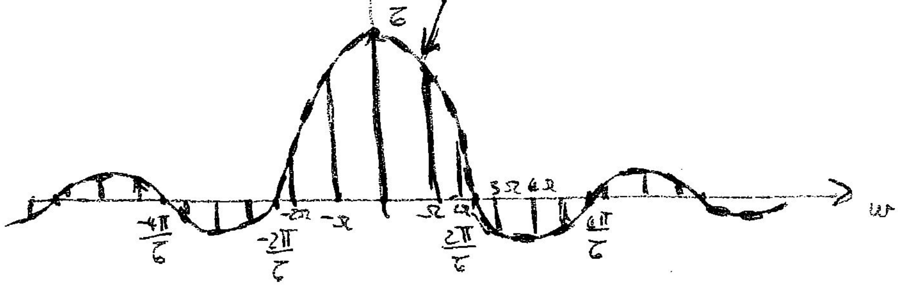
$\mathcal{F}\{\sum p_\tau(t-nT)\}$ $\qquad P_\tau(\omega) = \tau \operatorname{sinc} \frac{\omega\tau}{2}$ = anvelopa



= spectru discret, cu anvelopa $P_\tau(\omega)$

$$\Rightarrow \mathcal{F}\left\{ \sum p_\tau(t-nT) \right\} = \sum_{n=-\infty}^{\infty} P_\tau(n\cdot\Omega)\cdot\Omega\cdot \delta(\omega - n\Omega)$$

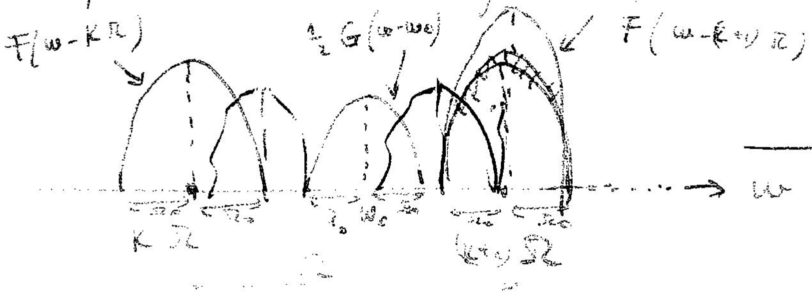
$$\Rightarrow Y(\omega) = \frac{1}{2\pi}\cdot F(\omega) * \left(\underset{=\frac{2\pi}{T}}{\Omega} \cdot \sum_{n=-\infty}^{\infty} \underbrace{P_\tau(n\Omega)}_{=\tau\cdot \operatorname{sinc}\frac{n\Omega\tau}{2}} \cdot \delta(\omega - n\Omega) \right) +$$

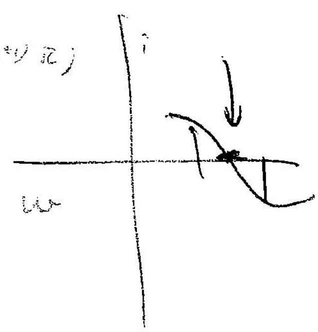
$$+ \frac{1}{2} G(\omega+\omega_0) + \frac{1}{2} G(\omega-\omega_0)$$

$$\Rightarrow Y(\omega) = \frac{1}{T}\cdot \sum_{n=-\infty}^{\infty} \tau \operatorname{sinc}\frac{n\Omega\tau}{2} \cdot F(\omega - n\Omega) + \frac{1}{2}G(\omega+\omega_0) + \frac{1}{2}G(\omega-\omega_0)$$

$F(\omega)$ ~~are banda~~
și $G(\omega)$ au banda limitata între $[-\Omega_0, \Omega_0]$

- Pt. ca $g(t)$ să poată fi recuperat: $\frac{1}{2}G(\omega+\omega_0)$ nu trebuie să se suprapună peste replicile lui $F(\omega)$ din jurul său:

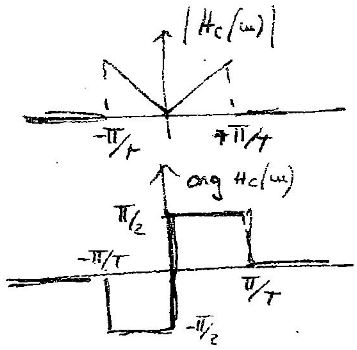




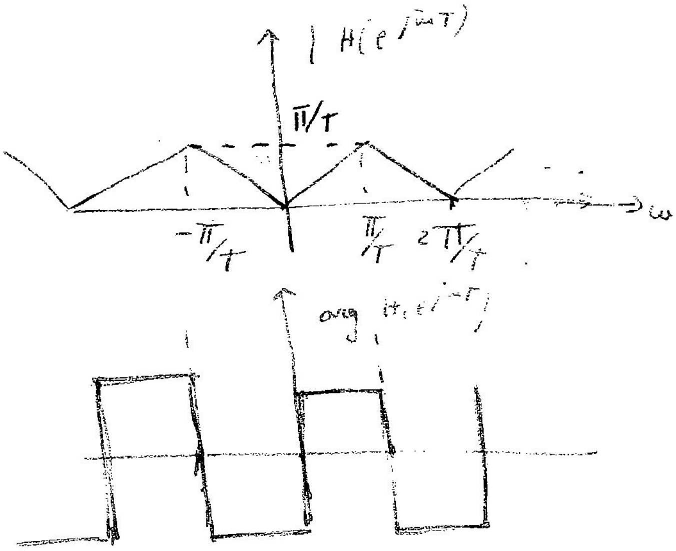
$$\Leftrightarrow \begin{cases} \dfrac{2\pi}{T} \geq 4\,\Omega_0 & \text{: eșantionarea cu cel puțin } \underline{\text{dublul frecv. Nyquist}} \\ & \text{(de 4 ori frecv. maximă)} \\ \text{Să existe } K \in \mathbb{N} \text{ a.î. } & K \cdot \underbrace{\dfrac{2\pi}{T}}_{\Omega} + 2\Omega_0 \leq \omega_0 \leq (K+1)\underbrace{\dfrac{2\pi}{T}}_{\Omega} - 2\Omega_0 \end{cases}$$

$\Leftrightarrow$ ω_0 situat convenabil

(5) $$H_c(\omega) = \begin{cases} j\omega & |\omega| < \dfrac{\pi}{T} \\ 0, & |\omega| \geq \dfrac{\pi}{T} \end{cases} \Rightarrow$$



$$H(e^{j\omega}) = \frac{j\omega}{T} \quad \text{pt. } |\omega| < \pi/T$$



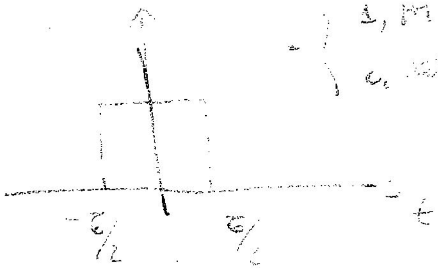
$h[n] = \mathcal{F}^{-1}\left\{\dfrac{H_c(\omega)}{\cancel{T}}\right\}$ eșantionat cu perioada T =

$= h(t)\big|_{t=nT}$, unde $h(t) = \mathcal{F}^{-1}\left\{\dfrac{H_c(\omega)}{\cancel{T}}\right\}$

$$\mathcal{F}^{-1}\left(\frac{H_c(\omega)}{\cancel{T}}\right) = \frac{1}{2\pi \cancel{T}}\int_{-\pi/T}^{\pi/T} j\omega \cdot e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} \frac{\partial\, e^{j\omega t}}{\partial t}$$

$h(t) \Longleftrightarrow \dfrac{H_c(\omega)}{\cancel{T}}$

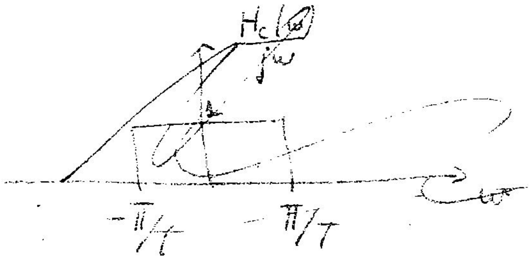
$=$



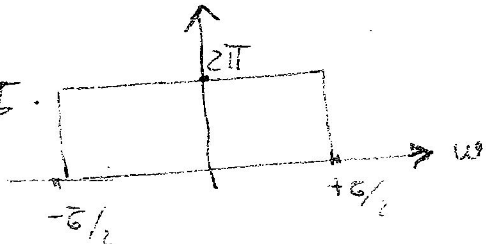
$$= \begin{cases} 1, & |t| < \tau/2 \\ 0, & \text{rest} \end{cases} \xleftrightarrow{\mathcal{F}} \tau \,\text{sinc}\, \frac{\omega \tau}{2}$$

=> Dualitate :

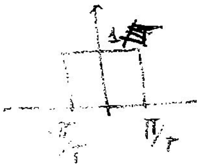
$$\frac{\pi}{\omega_0} = T \qquad \omega_0 = \frac{\pi}{T}$$



$$\tau \,\text{sinc}\, \frac{t \cdot \tau}{2} \xleftrightarrow{\mathcal{F}} 2\pi \cdot$$



$$\Rightarrow \frac{H_c(\omega)}{j\omega} =$$



$$\xrightarrow{\mathcal{F}^{-1}} \frac{1}{2\pi} \frac{2\pi}{T} \cdot \text{sinc}\, \frac{t \cdot \frac{2\pi}{T}}{2} = \frac{1}{T} \,\text{sinc}\, \frac{\pi}{T} t$$

$$\left(\omega_0 = \frac{\pi}{T} \Rightarrow \frac{\pi}{\omega_0} = \frac{\pi}{\frac{\pi}{T}} = T \right) \Rightarrow \int_0^t h(t) = \frac{1}{T} \,\text{sinc}\, \frac{\pi}{T} t$$

$$= \frac{1}{T} \cdot \frac{\sin \frac{\pi}{T} t}{\frac{\pi}{T} t} = \frac{1}{\pi \cdot t} \cdot \sin \frac{\pi}{T} \cdot t$$

$$\Rightarrow h(t) = \left(\frac{1}{\pi t} \cdot \sin \frac{\pi}{T} t \right)' = \frac{1}{\pi} \cdot \frac{-1}{t^2} \cdot \sin \frac{\pi}{T} t + \frac{1}{\pi t} \cdot \frac{\pi}{T} \cdot \cos \frac{\pi}{T} t$$

$$= \frac{1}{T \cdot t} \cdot \cos \frac{\pi}{T} t - \frac{1}{\pi t^2} \cdot \sin \frac{\pi}{T} t$$